Rekursif

Rekursif

- Proses yang memanggil dirinya sendiri.
- Merupakan suatu fungsi atau prosedur
- Terdapat suatu kondisi untuk berhenti.

Faktorial

- Konsep Faktorial
  n! = n(n-1)(n-2)…1
- Dapat diselesaikan dengan
  - Cara Biasa
  - Rekursif

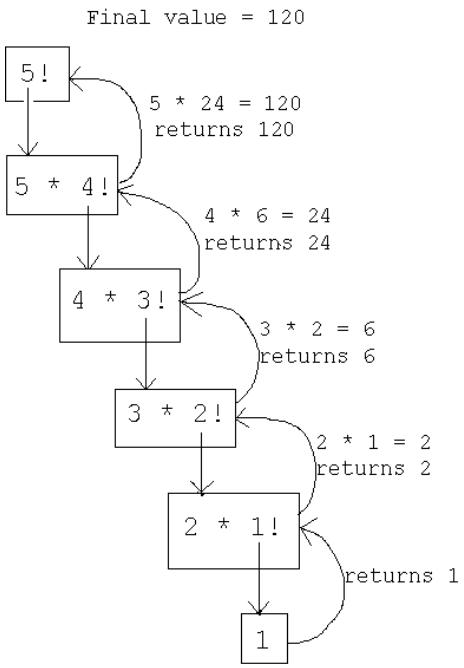
Iterative factorial:

$$n! = \prod_{k=1}^{n} k$$

Recursive factorial:

$$n! = \begin{cases} 1 & \text{if } n = 0, \\ (n-1)! \times n & \text{if } n > 0. \end{cases}$$

Faktorial



Faktorial : Cara Biasa

```
int Faktorial(int n)
{
    if (n<0) return -1 ;
    else if (n>1)
    {
         S = 1 ;
         for(i=2 ;i<=n;i++)  S = S * n ;
         return S ;
     }
     else return 1 ;
}
```

Faktorial dengan Rekursif

```
int Faktorial(int n)
{
     if (n<0) return -1;
     else if (n>1) return (n*Faktorial(n-1))
        else return 1 ;
}
```

Deret Fibonacci

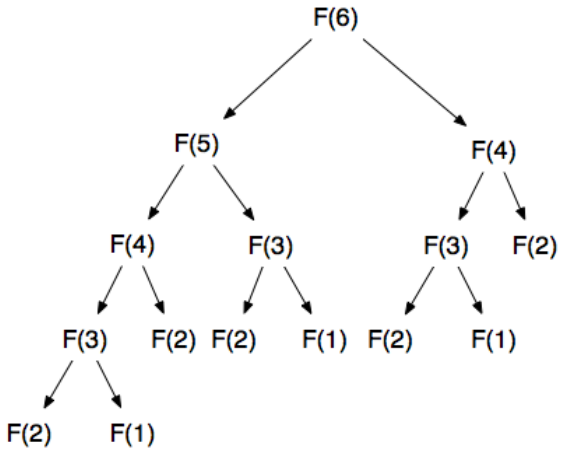
- Leonardo Fibonacci berasal dari Italia 1170-1250
- Deret Fibonacci f_1, f_2,… didefinisikan secara rekursif sebagai berikut :

  $f_1 = 1$

  $f_2 = 2$

  $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$
- Deret: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597,…

Deret Fibonacci



Deret Fibonacci

```
public class Fibonacci {
    public static long fib(int n) {
        if (n <= 1) return n;
        else return fib(n-1) + fib(n-2);
    }

    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        for (int i = 1; i <= N; i++)
            System.out.println(i + ": " + fib(i));
    }

}
```

Tower Hanoi

- Tower Hanoi adalah permainan puzzle dengan tiga tiang dan sejumlah disk/cakram yang tersusun pada tiang.
- Disk memiliki ukuran yang bervariasi. Disk-disk tertumpuk rapi berurutan berdasarkan ukurannya dalam salah satu tiang, disk terkecil diletakkan teratas, sehingga membentuk kerucut.
- Tujuan adalah untuk memasukkan semua disk dari tiang awal ke tiang tujuan dengan menggunakan tiang bantuan dengan aturan :
  - Hanya satu disk dapat dipindahkan pada suatu waktu
  - Sebuah disk tidak dapat ditempatkan di atas disk yang lebih kecil

1-10

The Towers of Hanoi puzzle

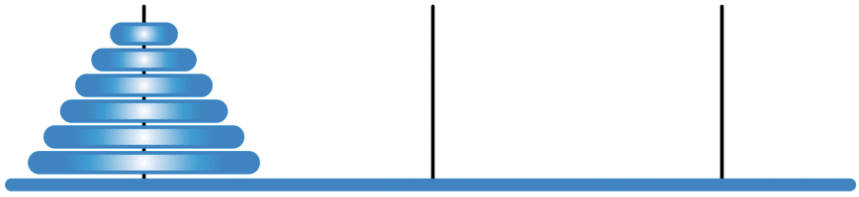


FIGURE 7.6 The Towers of Hanoi puzzle

1-11

A solution to the three-disk Towers of Hanoi puzzle

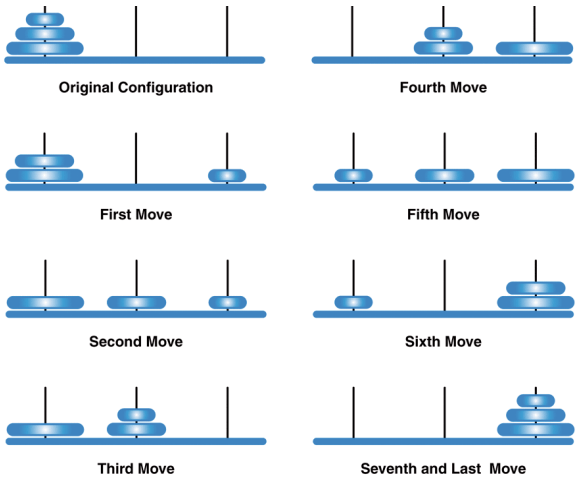
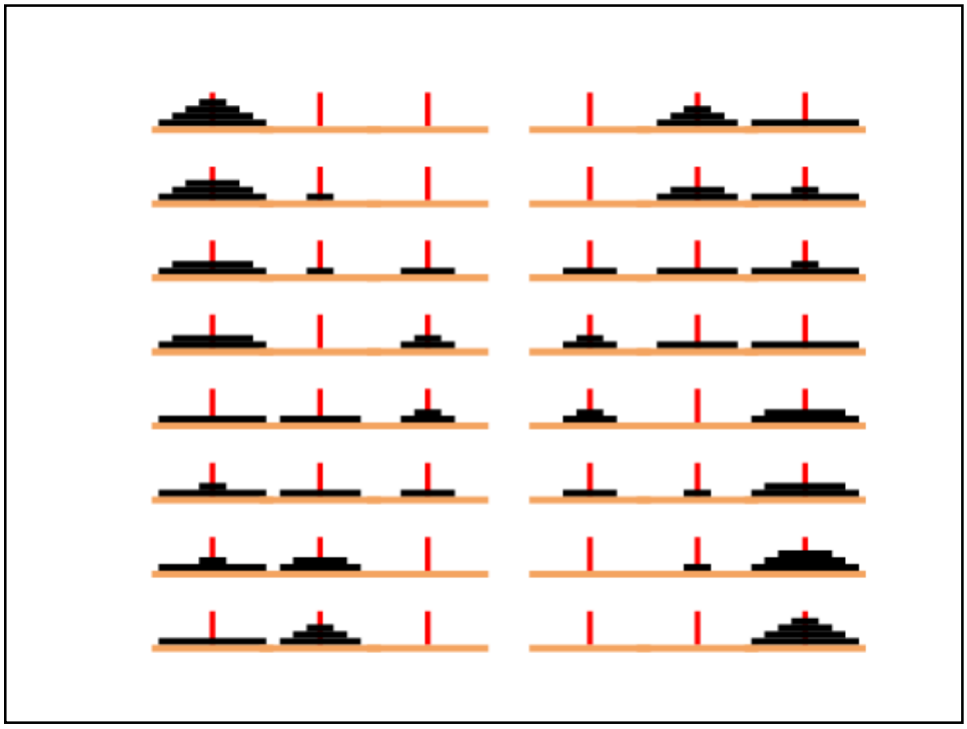
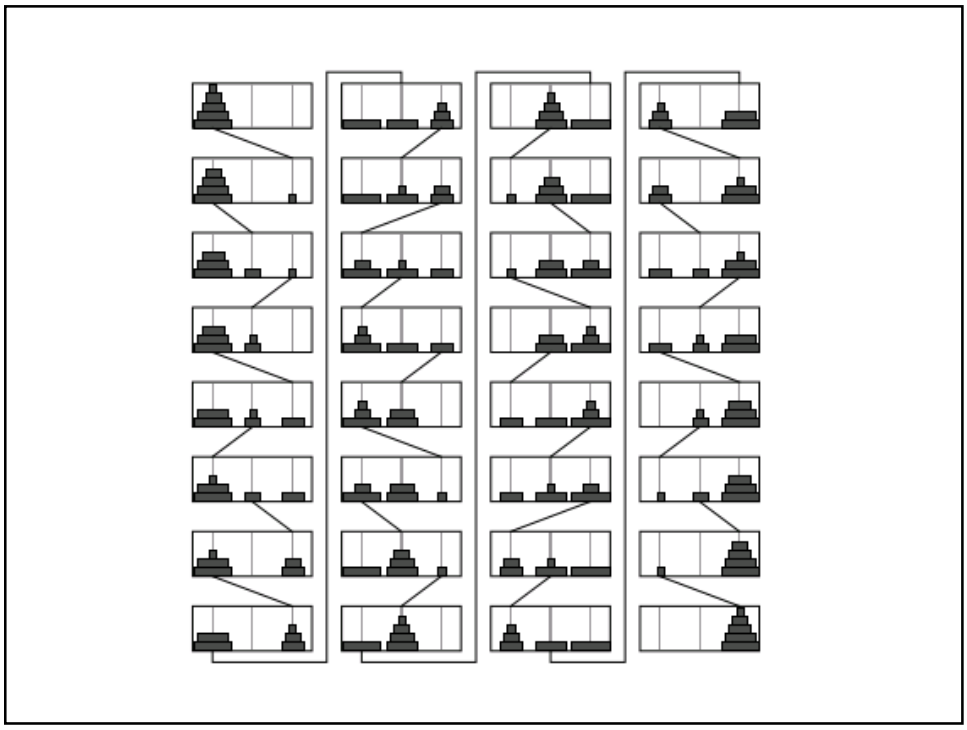


FIGURE 7.7 A solution to the three-disk Towers of Hanoi puzzle

1-12





Towers of Hanoi

- Untuk memindahkan N disk dari tiang asal ke tiang tujuan :
  - Pindahkan N-1 disk yang paling atas dari tiang asal ke tiang bantuan.
  - Pindahkan 1 disk terbesar dari tiang asal ke tiang tujuan
  - Pindahkan N-2 disk dari tiang bantuan ke tiang tujuan

1-15

Towers of Hanoi

- Jumlah perpindahan disk bertambah secara exponential dengan bertambahnya jumlah disk
- Solusi secara rekursif lebih sederhana dibandingkan dengan solusi iteratif

1-16

UML description of the SolveTowers and TowersofHanoi classes

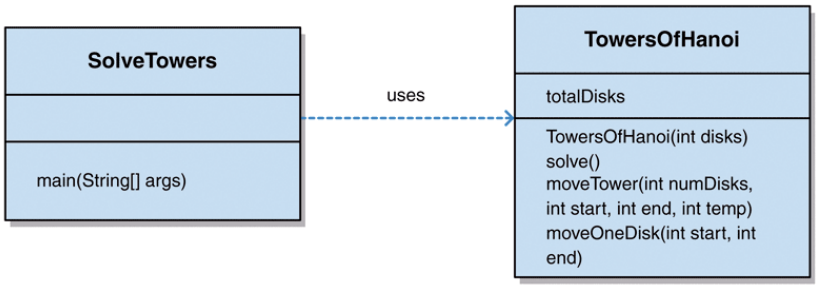


FIGURE 7.8 UML description of the `SolveTowers` and `TowersOfHanoi` classes

1-17

The Solve Towers class

```
/**
 * SolveTowers demonstrates recursion.
 *
 * @author Dr. Lewis
 * @author Dr. Chase
 * @version 1.0, 8/18/08
 */

public class SolveTowers
{
   /**
    * Creates a TowersOfHanoi puzzle and solves it.
    */
   public static void main (String[] args)
   {
      TowersOfHanoi towers = new TowersOfHanoi (4);
      towers.solve();
   }
}
```

1-18

The Towers of Hanoi class

```
/**
 * TowersOfHanoi represents the classic Towers of Hanoi puzzle.
 *
 * @author Dr. Lewis
 * @author Dr. Chase
 * @version 1.0, 8/18/08
 */

public class TowersOfHanoi
{
   private int totalDisks;

   /**
    * Sets up the puzzle with the specified number of disks.
    *
    * @param disks  the number of disks to start the towers puzzle with
    */
   public TowersOfHanoi (int disks)
   {
      totalDisks = disks;
   }
```

1-19

The Towers of Hanoi class (continued)

```
/**
   * Performs the initial call to moveTower to solve the puzzle.
   * Moves the disks from tower 1 to tower 3 using tower 2.
   */
  public void solve ()
  {
     moveTower (totalDisks, 1, 3, 2);
  }

  /**
   * Moves the specified number of disks from one tower to another
   * by moving a subtower of n-1 disks out of the way, moving one
   * disk, then moving the subtower back. Base case of 1 disk.
   *
   * @param numDisks  the number of disks to move
   * @param start     the starting tower
   * @param end       the ending tower
   * @param temp      the temporary tower
   */
```

1-20

The Towers of Hanoi class (continued)

```
private void moveTower (int numDisks, int start, int end, int temp)
  {
    if (numDisks == 1)
      moveOneDisk (start, end);
    else
    {
      moveTower (numDisks-1, start, temp, end);
      moveOneDisk (start, end);
      moveTower (numDisks-1, temp, end, start);
    }
  }

  /**
   * Prints instructions to move one disk from the specified start
   * tower to the specified end tower.
   *
   * @param start  the starting tower
   * @param end    the ending tower
   */
  private void moveOneDisk (int start, int end)
  {
    System.out.println ("Move one disk from " + start + " to " +
                        end);
  }
}
```

1-21

Multibase Representations

- Decimal is only one representation for numbers. Other bases include 2 (binary), 8 (octal), and 16 (hexadecimal).
  - Hexadecimal uses the digits 0-9 and a=10, b=11, c=12, d=13, e=14, f=16.

$95 = 1011111_2$ // 95 $= 1(2^6)+0(2^5)+0(2^4)+0(2^3)+1(2^2)+1(2^1)+1(2^0)$
$= 1(64)+0(32)+1(16)+1(8)+1(4)+1(2)+1$

$95 = 340_5$ // 95 $= 3(52) + 4(51) + 0(5^0)$
$= 3(25) + 4(5) + 0$

$95 = 137_8$ // 95 $= 1(82) + 3(81) + 7(8^0)$
$= 1(64) + 3(8) + 7$

$748 = 2ec_{16}$ // 748 $= 2(16^2) + 14(16^1) + 12(16^0)$
$= 2(256) + 14(16) + 12$
$= 512 + 224 + 12$

Multibase Representations (continued)

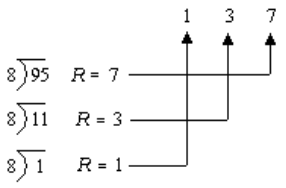
- An integer n > 0 can be represented in different bases using repeated division.
  - Generate the digits of n from right to left using operators '%' and '/'. The remainder is the next digit and the quotient identifies the remaining digits.

Multibase Representations (continued)

Step 1: $8\overline{)95}$ quotient 11, remainder 7 (88)

Step 2: $8\overline{)11}$ quotient 1, remainder 3 (8)

Step 3: $8\overline{)1}$ quotient 0, remainder 1 (0)




```
85 % 8 = 7       // remaining digits: 85/8 = 11
11 % 8 = 3       // remaining digits: 11/8 = 1
 1 % 8 = 1       // remaining digits: 1/8 = 0
   0             // stopping condition
```

Multibase Representations (continued)

- Convert n to base b by converting the smaller number n/b to base b (recursive step) and adding the digit n%b.

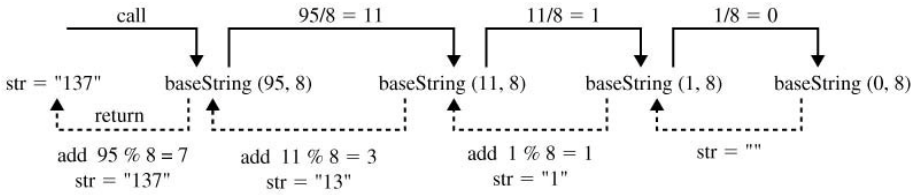
Multibase Representations (continued)

```
// returns string representation
// of n as a base b number
public static String baseString(int n, int b)
{
   String str = "", digitChar = "0123456789abcdef";

   // if n is 0, return empty string
   if (n == 0)
      return "";
   else
   {
      // get string for digits in n/b
      str = baseString(n/b, b);   // recursive step

      // return str with next digit appended
      return str + digitChar.charAt(n % b);
   }
}
```

Multibase Representations (concluded)



Series of recursive method calls for baseString(95,8).

Rekursif Tail

- Jika hasil akhir yang akan dieksekusi berada dalam tubuh fungsi
- Tidak memiliki aktivitas selama fase balik.

What is Tail Recursion?

- Metode rekursif
  - Tail recursive
  - Nontail recursive
- Method tail recursive memiliki pemanggilan rekursif di akhir method.
- Recursive methods yang bukan tail recursive disebut non-tail recursive

Is Factorial Tail Recursive?

- Apakah method faktorial adalah tail recursive ?

```
int fact(int x){
   if (x==0)
      return 1;
   else
      return x*fact(x-1);
}
```

- Bukan tail recursive karena pada saat kembali dari recursive call `x*fact(x-1)`, masih terdapat operasi perkalian.

Another Example

- Apakah method tail() adalah tail recursive?

```
void tail(int i) {
if (i>0) {
   system.out.print(i+"")
tail(i-1)
}
```

- Method tail() merupakan tail recursive

Third Example

- Apakah method prog() adalah tail recursive?

```
void non prog(int i) {
   if (i>0) {
         prog(i-1);
         System.out.print(i+"");
         prog(i-1);
   }
}
```

- Tidak, karena dibaris awal terdapat recursive call
- Pada tail recursive, recursive call menjadi statement akhir, dan tidak ada recursive call diatasnya.

Advantage of Tail Recursive Method

- Method dengan Tail Recursive, mudah dirubah menjadi iteratif

```
void tail(int i){
  if (i>0) {
    system.out.println(i+"");
    tail(i-1)
  }
}
```

```
void iterative(int i){
  for (;i>0;i--)
    System.out.println(i+"");
}
```

- Smart compilers can detect tail recursion and convert it to iterative to optimize code
- Used to implement loops in languages that do not support loop structures explicilty (e.g. prolog)

Deret Fibonacci

```
public class Fibonacci {
    public static long fib(int n) {
        if (n <= 1) return n;
        else return fib(n-1) + fib(n-2);
    }

    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        for (int i = 1; i <= N; i++)
            System.out.println(i + ": " + fib(i));
    }

}
```

Converting Non-tail to Tail Recursive

- Method non-tail recursive dapat diubah menjadi tail-recursive method dengan menambahkan parameter tambahan untuk menampung hasil.

  `method fact(int n)` → `fact_aux(int n, int result)`
- Teknik yang digunakan biasanya membuat fungsi tambahan `(method fact(int n))` yang memanggil method tail recursive.

```
int fact_aux(int n, int result) {
   if (n == 1)
      return result;
   return fact_aux(n - 1, n * result)
}
int fact(n) {
   return fact_aux(n, 1);
}
```

Rekursif Tail : Faktorial()

$$F(n,a) = \begin{cases} a & \text{jika } n=0,\ n=1 \\ F(n-1,na) & \text{jika } n>1 \end{cases}$$

F(4,1) = F(3,4)	Fase awal
F(3,4) = F(2,12)	
F(2,12) = F(1,24)	
F(1,24) = 24	Kondisi Terminal
24	Fase Balik Rekursif Lengkap

Converting Non-tail to Tail Recursive

- Method tail-recursive Fibonacci diimplementasikan menggunakan dua parameter bantuan untuk menampung hasil.

auxiliary parameters!

```
int fib_aux ( int n , int next, int result)
{
  if (n == 0)
    return result;
  return fib_aux(n - 1, next + result, next);
}
```

To calculate fib(n) , call fib_aux(n,1,0)